Precision Measurement of Circular Polarization States of Light with Rotating Wave-plate Polarimeter

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Abstract

Stokes parameters corresponding to nearly perfect left- and right-circularly polarized light are measured with three significant figures using a home-made rotating wave-plate polarimeter. The high precision and simplicity of the polarimeter can find use in many fields of basic science and industrial applications, where precise determination of polarization status is highly desirable.

Introduction

Precise measurement of polarization state of light is required in various fields of modern science and engineering. Especially, circularly polarized beams have been used for the studies of laser cooling and trapping of atoms [1], measurements of Faraday rotation, molecular chirality, and quantum information science exploiting the orbital angular momentum of light. Since the early work of Stokes [2], numerous methods of measuring light polarization state have been suggested, and among them, a simple but precise **rotating wave-plate polarimeter** (RWP) was proposed recently by V. Andreev [3], which has a capability of self-calibration to reduce the measurement uncertainty. We present our home-built RWP to measure precisely the polarization state of nearly perfect left- and right-circularly polarized light from a single-frequency Nd:YAG laser at 1064 nm. We plan to apply the RWP in quantum optics experiments such as precise measurement of “mode polarization” proposed recently to explain the Bohr’s complementarity theory without any missing “hidden polarization” in optical interferometer [4] with a single-photon source.

Theoretical descriptions

- Müller calculus : $(I_2, M_2, C_2, S_2)^T = \mathrm{M}_{\text{linear polarizer}}\mathrm{M}_{\text{waveplate}}(I_1, M_1, C_1, S_1)^T$

$$\Rightarrow I_{out}(I_1, M_1, C_1, S_1) = I_{out}(\tilde{\alpha}, \tilde{\beta}) = C_0 + C_2\cos(2\tilde{\beta}) + C_4\cos(4\tilde{\beta}) + S_2\sin(2\tilde{\beta}) + S_4\sin(4\tilde{\beta})$$

- Fourier coefficients : $\begin{cases} \tilde{\alpha} = 0 & : C_0, C_2, C_4, S_2, S_4 \\ \tilde{\alpha} = \pi/4 & : \tilde{C}_0, \tilde{C}_2, \tilde{C}_4, \tilde{S}_2, \tilde{S}_4 \end{cases}$

- Calibration angles :

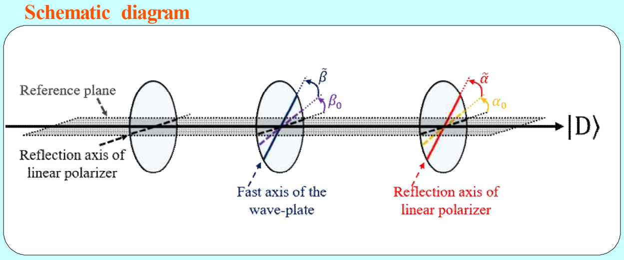
$$\begin{cases} \alpha_0 = -\dfrac{1}{2} - \dfrac{C_0\tilde{S}_4 - \tilde{C}_0 C_4}{\left(\sqrt{C_4^2 + S_4^2} - C_0\right)\tilde{S}_4}, \quad \beta_0 = \dfrac{1}{4}\left(\arctan\dfrac{S_4}{C_4} - 2\alpha_0\right) \\ \delta = \dfrac{\pi}{2} + 1 + 2\dfrac{\sqrt{C_4^2 + S_4^2} - C_0}{\sqrt{C_4^2 + S_4^2} + C_0} \end{cases}$$

- Input Stokes parameters :

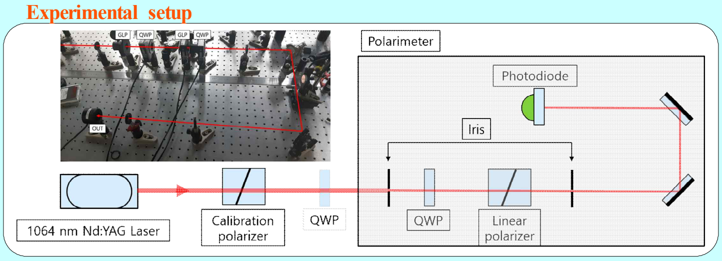
$$\begin{cases} I = C_0 - (1+\cos\delta)(1-\cos\delta)^{-1}\left[C_4\cos(4\alpha_0 + 4\tilde{\alpha} - 4\beta_0) + S_4\sin(4\alpha_0 + 4\tilde{\alpha} - 4\beta_0)\right] \\ M = 2(1-\cos\delta)^{-1}\left[C_4\cos(2\alpha_0 + 2\tilde{\alpha} - 4\beta_0) + S_4\sin(2\alpha_0 + 2\tilde{\alpha} - 4\beta_0)\right] \\ C = 2(1-\cos\delta)^{-1}\left[S_4\cos(2\alpha_0 + 2\tilde{\alpha} - 4\beta_0) - C_4\sin(2\alpha_0 + 2\tilde{\alpha} - 4\beta_0)\right] \\ S = C_2\left[\sin(\delta)\sin(2\alpha_0 + 2\tilde{\alpha} - 2\beta_0)\right]^{-1} = -S_2\left[\sin(\delta)\cos(2\alpha_0 + 2\tilde{\alpha} - 2\beta_0)\right]^{-1} \end{cases}$$

Schematic diagram & Experimental setup

Schematic diagram



Experimental setup

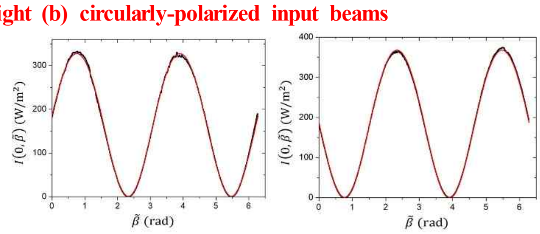


Experimental Results

Measured Stokes and Jones vector components

Left-circular polarization state		
	Theory	Experiment
Stokes vector	$\begin{bmatrix} M/I \\ C/I \\ S/I \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$	$\begin{bmatrix} 0.0653 \pm 0.0013 \\ -0.0219 \pm 0.0014 \\ -0.9928 \pm 0.0010 \end{bmatrix}$
	Theory	Experiment
Jones vector	$\begin{bmatrix} 1 \\ i \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0.0207 \pm 0.0013 \\ + (0.9361 \pm 0.0010)\ i \end{bmatrix}$
Right-circular polarization state		
	Theory	Experiment
Stokes vector	$\begin{bmatrix} M/I \\ C/I \\ S/I \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	$\begin{bmatrix} 0.061\ 60 \pm 0.000\ 96 \\ -0.0188 \pm 0.0010 \\ 0.998\ 86 \pm 0.000\ 76 \end{bmatrix}$
	Theory	Experiment
Jones vector	$\begin{bmatrix} 1 \\ -i \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0.017\ 70 \pm 0.000\ 98 \\ - (0.940\ 02 \pm 0.00\ 91)\ i \end{bmatrix}$

Experimental (black) and fitted (red) curves of $I(0, \tilde{\beta})$ for left (a) and right (b) circularly-polarized input beams

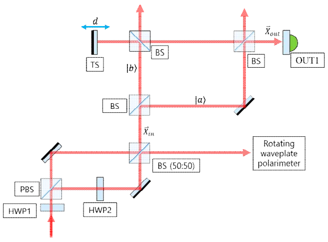


Measured self-calibration angles $(\alpha_0, \beta_0, \delta)$

Glan-laser polarizer reflection-axis offset angle (α_0)	$(0.75 \pm 0.13)^\circ$
Wave-plate fast-axis offset angle (β_0)	$(-2.787 \pm 0.080)^\circ$
Wave-plate retardation angle (δ)	$(90.199 \pm 0.068)^\circ$

Conclusion and outlook

We have measured the polarization states corresponding to perfect left- and right-circularly polarized laser beams, i.e., the components of relative Stokes vectors and normalized Jones vectors with a measurement uncertainty of 0.001 by using a home-made RWP. The RWP will be used in the experiment below to confirm the polarization coherence theorem [4] for light with non-separable degrees of freedom.



$$|\Phi> = \cos\theta|a>|s_a> + \sin\theta\, e^{i\phi}|b>|s_b>$$

$$W_s = (<E_j^*(t)E_k(t)>),\ j,k \in a,b;$$
$$P = \lambda_+ - \lambda_-;\ C = 2\sqrt{\lambda_+\lambda_-}$$

$$\text{Concurrence}: \mathrm{C}(\Phi) = |<\Phi|\tilde{\Phi}>|$$
$$P = \sqrt{S_{1,(a,b)}^2 + S_{2,(a,b)}^2 + S_{3,(a,b)}^2}$$
$$C^2 + P^2 = 1;\ \text{Polarization coherence theorem}$$

References

[1] W. D. Phillips, Laser cooling and trapping of neutral atoms. Rev. Mod. Phys. **70**, 721 (1998).
[2] G. Stokes, *Mathematical and Physical Papers, Vol.* 3 (Cambridge University, 1901).
[3] V. Andreev, A self-calibrating polarimeter to measure Stokes parameters, arXiv:1703.00963 (2017).
[4] J. Eberly, Polarization coherence theorem, Opt. **4**(9), 1113-1114 (2017).