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Analytic framework for understanding the competing multiple light scattering processes

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1. Motivation – find the working condition of the proposed method

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Focusing of light energy inside a scattering medium by controlling the time-gated multiple light scattering

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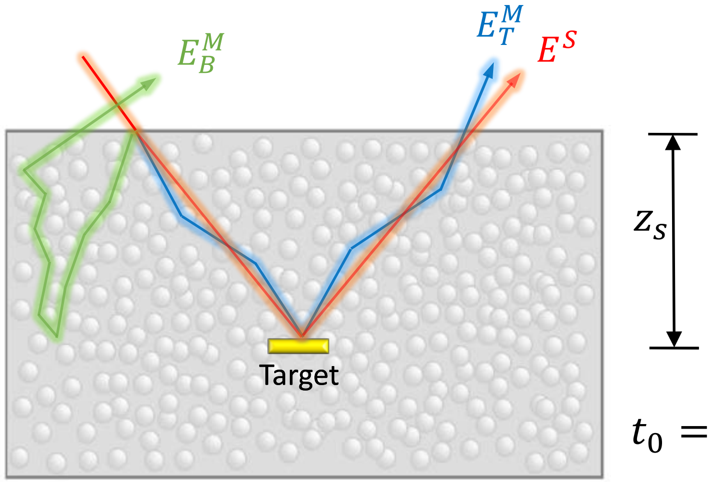
2. Summary of the method

Time-gated eigenchannel coupling

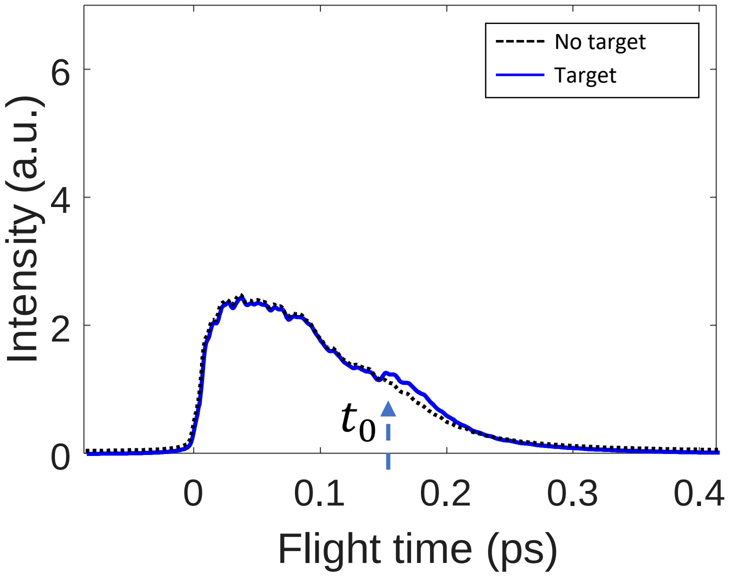
- find eigenchannels of R in order to maximize the total backscattering intensity set by R
- maximizing the total intensity → enhancing target energy delivery?

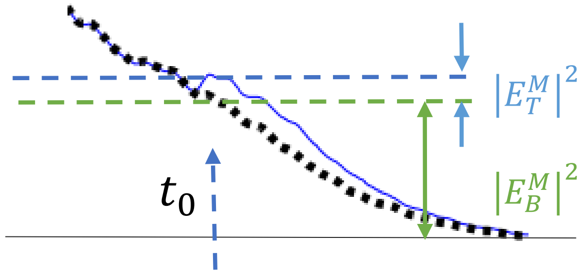
even when $\left|E_T^M\right|^2 < \left|E_B^M\right|^2$



$$t_0 = \frac{2z_s}{c}$$

$$E_R(t_0) = E^S + \boxed{E_T^M} + E_B^M$$





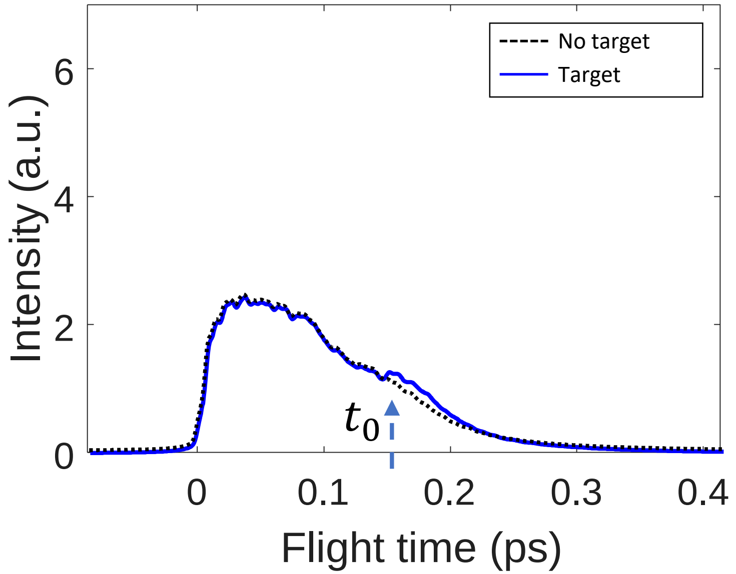
Principle – two important parameters

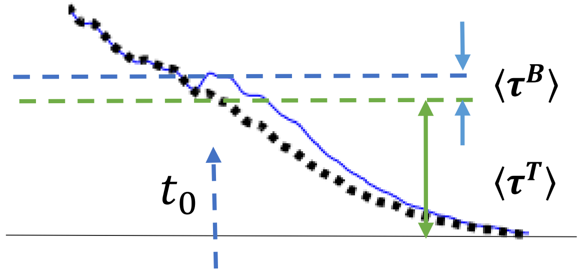
$R \approx M_T + M_B,$ $\rightarrow$ interplay between M_T and M_B in determining $\boldsymbol{v}_1^R$.

where M_T and M_B are constructed from E_T^M and E_B^M.

1. average eigenvalue, $\langle\boldsymbol{\tau}^T\rangle$ and $\langle\boldsymbol{\tau}^B\rangle$ $\rightarrow \langle\boldsymbol{\tau}^B\rangle/\langle\boldsymbol{\tau}^T\rangle$

- the **average intensities** of the target and background multiple-scattered waves.





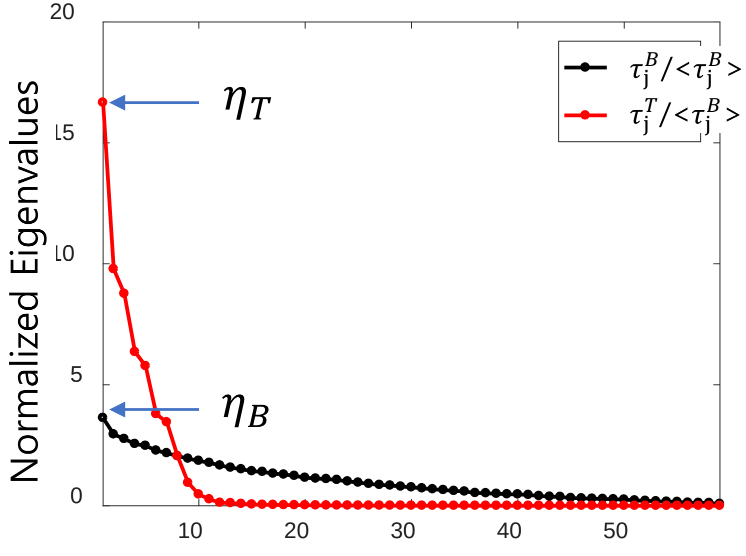
Principle – two important parameters

2. enhancement factors, $\boldsymbol{\eta_T}$ and $\boldsymbol{\eta_B}$ $\rightarrow \boldsymbol{\eta_B / \eta_T}$

- $\eta_T = \tau_1^T / \langle \tau^T \rangle$, $\eta_B = \tau_1^B / \langle \tau^B \rangle$
- the **effectiveness** of the wavefront control
- $\boldsymbol{\eta_T > \eta_B}$ for targets smaller than the field of view:

  reduced number of effective channels

  → increased correlation among channels. [1]



Eigenchannel index (*j*)

If $\boldsymbol{\eta_T > \eta_B}$ and $\langle \boldsymbol{\tau^T} \rangle = \langle \boldsymbol{\tau^B} \rangle$,

light wave **injected to $\boldsymbol{v_1^R}$** will preferably couple its energy to M_T rather than M_B

because $\boldsymbol{\eta_T > \eta_B}$.

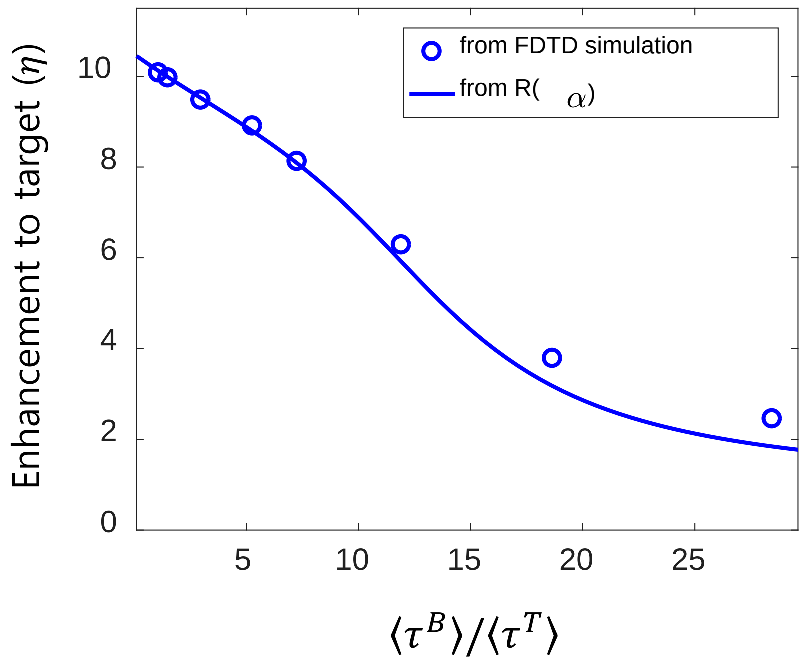
➡ ***preferable*** **enhancement of light energy delivered to the target object.**

[1] C. W. Hsu et al., Nat Phys 13, 497 (2017)

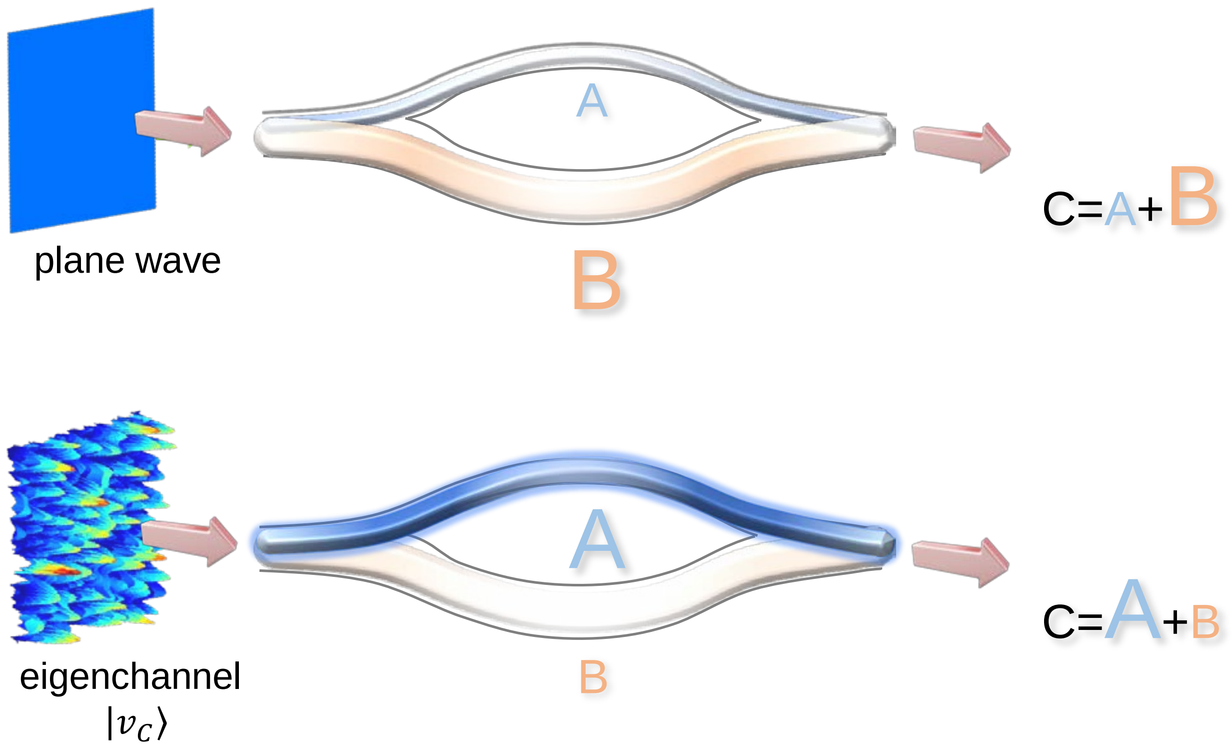
3. Goal: working condition

We empirically found the working condition of the method from numerically simulations in the previous study.

Can we analytically obtain the working condition only using the two important parameters?

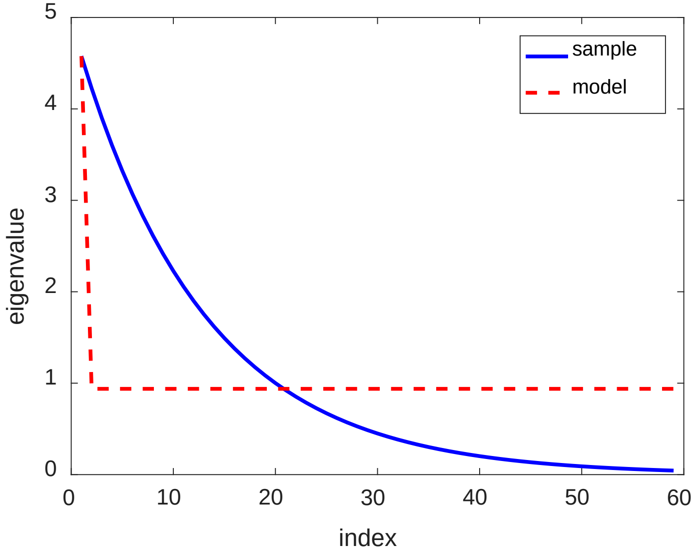


4. Generalized problem



Find $|v_C\rangle$ maximizing total output energy (E_C) given $\langle\tau^A\rangle$, $\langle\tau^B\rangle$, η_A and η_B.

5. Assumption



$$|v_C\rangle = \sqrt{\alpha_A} e^{i\phi_{A,1}} |v_{A,1}\rangle + \sum_{i=2}^{N} \sqrt{\frac{1-\alpha_A}{N-1}} e^{i\phi_{A,i}} |v_{A,i}\rangle$$

$$|v_C\rangle = \sqrt{\alpha_B} e^{i\phi_{B,1}} |v_{B,1}\rangle + \sum_{i=2}^{N} \sqrt{\frac{1-\alpha_B}{N-1}} e^{i\phi_{B,i}} |v_{B,i}\rangle$$

Find α_A, $\phi_{A,i}$, α_B, and $\phi_{B,i}$ that maximize E_C.

6. Derivation

Eliminating variables

$$E_C = \langle v_C|C^+C|v_C\rangle \cong \langle v_C|A^+A|v_C\rangle + \langle v_C|B^+B|v_C\rangle$$

$$\left\{\begin{aligned} |v_C\rangle &= \sqrt{\alpha_A}e^{i\phi_{A,1}}|v_{A,1}\rangle + \sum_{i=2}^{N}\sqrt{\frac{1-\alpha_A}{N-1}}e^{i\phi_{A,i}}|v_{A,i}\rangle \\ |v_C\rangle &= \sqrt{\alpha_B}e^{i\phi_{B,1}}|v_{B,1}\rangle + \sum_{i=2}^{N}\sqrt{\frac{1-\alpha_B}{N-1}}e^{i\phi_{B,i}}|v_{B,i}\rangle \end{aligned}\right.$$

$$E_C = \frac{N}{N-1}\langle\tau_A\rangle[(\eta_A-1)\boldsymbol{\alpha_A} - \eta_A/N + 1] + \frac{N}{N-1}\langle\tau_B\rangle[(\eta_B-1)\boldsymbol{\alpha_B} - \eta_B/N + 1]$$

Constraint

$$\boxed{|v_C\rangle(|v_{A,i}\rangle) = |v_C\rangle(|v_{B,i}\rangle)}$$

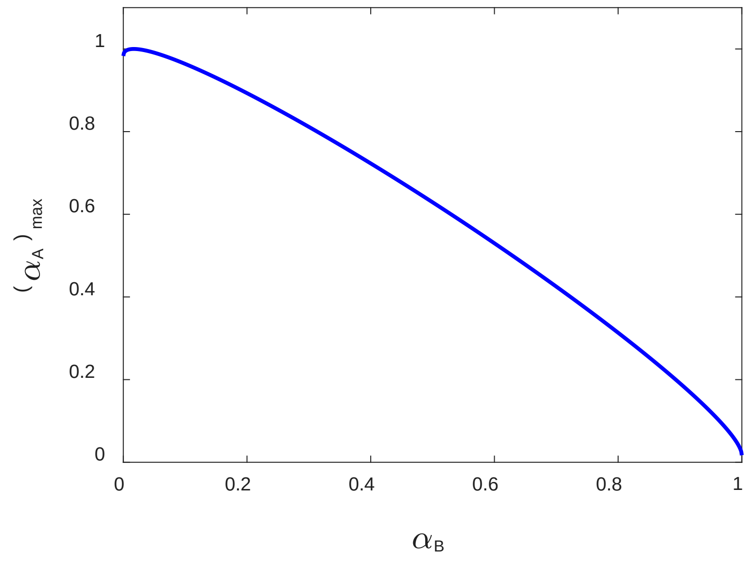
if A and B are independent

$$|v_{B,j}\rangle = \frac{1}{\sqrt{N}}\sum_{i=1}^{N} e^{i\theta_{ji}}|v_{A,i}\rangle$$

$$\alpha_A = \frac{1}{N}\left|\sqrt{\alpha_B}e^{i\phi_{B,1}}e^{i\theta_{11}} + \frac{\sqrt{1-\alpha_B}}{\sqrt{N-1}}\left(\sum_{i=2}^{N} e^{i\phi_{B,i}}e^{i\theta_{i1}}\right)\right|^2$$

find $e^{i\phi_{B,i}}$ maximizing α_A

$$(\alpha_A)_{\max} = \frac{1}{N}\left(\sqrt{\alpha_B} + \sqrt{N-1}\sqrt{1-\alpha_B}\right)^2$$



Find α_B maximizing E_C

$$E_C = \frac{N}{N-1}\langle\tau_A\rangle[(\eta_A - 1)\boldsymbol{\alpha_A} - \eta_A/N + 1] + \frac{N}{N-1}\langle\tau_B\rangle[(\eta_B - 1)\boldsymbol{\alpha_B} - \eta_B/N + 1]$$

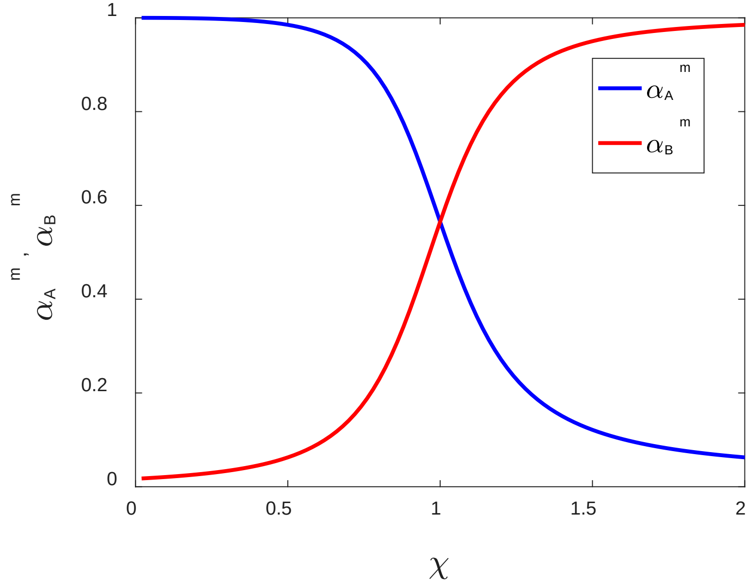
$$(\alpha_A)_{\max} = \frac{1}{N}\left(\sqrt{\alpha_B} + \sqrt{N-1}\sqrt{1-\alpha_B}\right)^2 \quad \Rightarrow \quad E_C(\alpha_B)$$

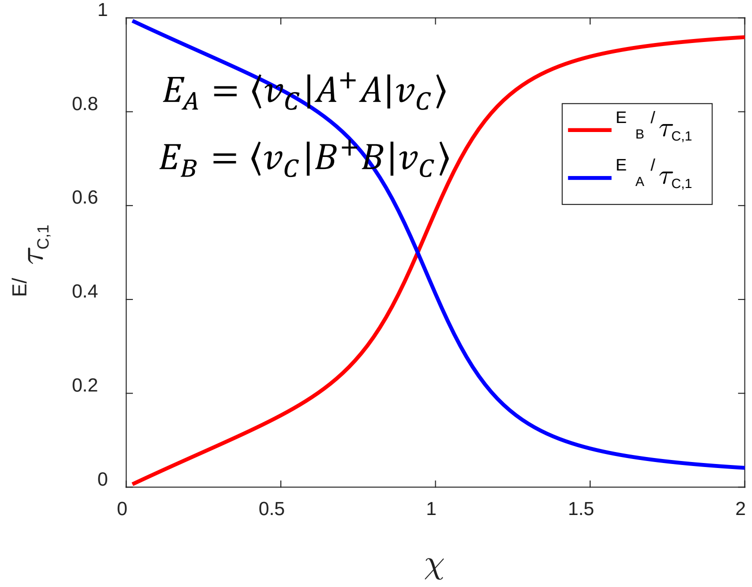
$$\boxed{\frac{dE_C(\alpha_B)}{d\alpha_B} = 0}$$

$$\alpha_B^m \approx \frac{1}{2} + \frac{1}{2}\frac{\chi - 1}{\sqrt{\frac{4}{N} + (\chi - 1)^2}}$$

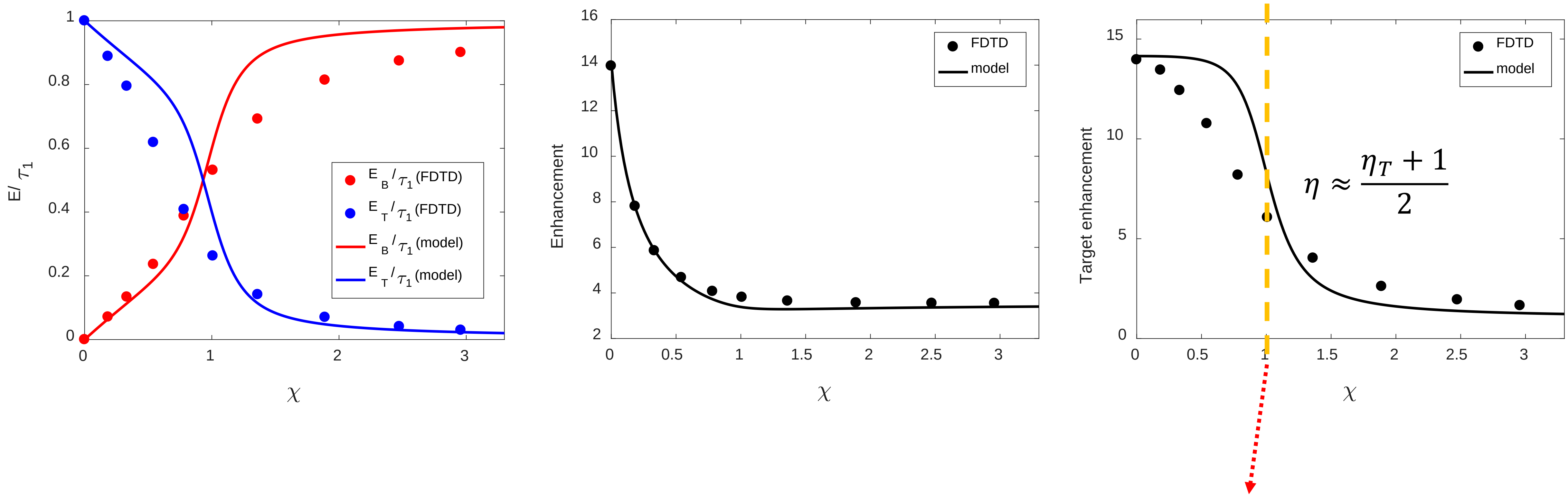
$$\alpha_A^m \approx \frac{1}{2} + \frac{1}{2}\frac{\frac{1}{\chi} - 1}{\sqrt{\frac{4}{N} + \left(\frac{1}{\chi} - 1\right)^2}}$$

$$\left(\chi \equiv \frac{\eta_B - 1}{\eta_A - 1}\frac{\langle\tau_B\rangle}{\langle\tau_A\rangle}\right)$$





7. Validity test with FDTD results



the working condition: $\chi < 1$

8. Summary

- Developed analytic framework for understanding the preferable coupling

- Found that $\chi \equiv \frac{(\eta_B - 1)\,\langle\tau_B\rangle}{(\eta_A - 1)\,\langle\tau_A\rangle}$ is a governing parameter that determines which subprocess is preferable at the time of coupling waves to the eigenchannel of C with the largest eigenvalue.

- Found the working condition, $\chi < 1$.

- The validity of the derived equation was supported by the exact solutions of the transfer matrices from the FDTD simulations.

- This work provides a new analytic framework for understanding the effect of wavefront shaping in the control of wave propagation in disordered media.