Spectral graph theory: Four common spectra

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Abstract

Spectral graph theory studies connections between combinatorial properties of graphs and the eigenvalues of matrices associated to the graph, such as the adjacency matrix, the Laplacian matrix, the signless Laplacian matrix and the normalized Laplacian matrix. Let $G = (V, E)$ be a simple graph. Denote by $D(G)$ the diagonal matrix of its vertex degrees and by $A(G)$ its adjacency matrix. Then the Laplacian matrix of G is $L(G) = D(G) - A(G)$. Denote the spectrum of $L(G)$ by $S(L(G)) = (\mu_1, \mu_2, \ldots, \mu_n)$, where we assume the eigenvalues to be arranged in nonincreasing order: $\mu_1 \geq \mu_2 \geq \cdots \geq \mu_{n-1} \geq \mu_n = 0$. Let a be the algebraic connectivity of graph G. Then $a = \mu_{n-1}$. Among all eigenvalues of the Laplacian matrix of a graph, the most studied is the second smallest, called the algebraic connectivity (a(G)) of a graph [5]. In this talk we will introduce the basics of spectral graph theory, we will show some results on $\mu_1(G)$ and $a(G)$ of graph G. We obtain some integer and real Laplacian (adjacency, signless Laplacian, normalized) eigenvalues of graphs. Moreover, we discuss several relations between eigenvalues (adjacency, Laplacian, signless Laplacian) and graph parameters. Finally, we give some conjectures on the spectral graph theory.

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